Optimal Zoning in Spatial Differentiation*

Diferenciación Espacial y Zonificación Óptima

Hamid Hamoudi**
Isabel Rodríguez***
Marcos Sanz Martín-Bustamante****

Abstract

This study analyzes optimal zoning policy in a duopolistic spatial competition framework for both circular and linear spaces. A regulator is introduced in the third stage of the price-location game through a welfare function to model zoning preferences from firms and consumers. An equilibrium outcome is then found for both spatial configurations. When the regulator is inclined to favor consumers (consumer-oriented) both firms are restricted to locate at one point to serve the whole market. Nevertheless, when the preferences of the regulator are biased towards firms (firm-oriented) the zoning area is maximized, with both firms being located at the market boundaries. The equilibrium outcome confirms location patterns found in real life situations under a non-neutral regulator.

Key words: *Zoning, spatial competition, welfare function, equilibrium results, industrial policy.*

JEL Classification: *C72, D47, D60, L51, L13.*

Resumen

Este estudio analiza las políticas de zonificación óptima en un marco de competencia espacial duopolística para los espacios lineal y circular. En la tercera etapa del juego de localización-precio se introduce un regulador que optimiza una función de bienestar que formaliza las preferencias de empresas y consumidores.

* We thank the editor of *Estudios de Economía* and two anonymous reviewers for very helpful comments and suggestions on the original manuscript. This article has received financial support from: "Proyecto de Excelencia Investigadora: *Análisis Económico de Políticas Públicas*" (URJC-BSCH).

** Universidad Rey Juan Carlos de Madrid. Paseo Artilleros s/n. Madrid 28032. Fundamentos de Análisis Económico. Email: hamid.hamoudi@urjc.es.

*** Universidad Rey Juan Carlos de Madrid. Paseo Artilleros s/n. Madrid 28032. Fundamentos de Análisis Económico. Email: Isabel.rodriguez@urjc.es.

**** [Corresponding autor] Universidad Rey Juan Carlos de Madrid. Paseo Artilleros s/n. Madrid 28032. Fundamentos de Análisis Económico. Email: marcos.sanz.martinbustamante@urjc.es.

Se obtiene el resultado de equilibrio para ambas configuraciones espaciales. Cuando el regulador se siente inclinado a favorecer a los consumidores (orientación al consumidor), ambas empresas restringen su localización a un mismo punto para atender el mercado entero. Sin embargo, cuando las preferencias del regulador están sesgadas hacia las empresas (orientación empresarial) se maximiza el área zonificada y ambas empresas se sitúan en los extremos del mercado. Este resultado ilustra ejemplos reales en presencia de autoridades reguladoras no neutrales.

Palabras clave: *Zonificación, competencia espacial, función de bienestar, resultado de equilibrio, política industrial.*

Clasificación JEL: *C72, D47, D60, L51, L13*

1. Introduction

In this study a regulator designing an urban area divides the space for two uses: an exclusively residential area for consumers (residents) and a hybrid area where consumers and firms may locate. The aim of the regulator when restricting certain areas is to provide a high-quality environment, reduce trouble and reduce local crime.

Cities and urban contexts provide many real examples of externality effects from firms on the rest of society. In some cases public intervention may become an internalization mechanism for negative externalities such as pollution or lack of residential space. Regulators may decide on the size of the restricted area depending on different factors. For instance, historical cities with a well-preserved city centers sometimes introduce specific regulations for firms locating in the old town area. Authorities may seek to preserve quality space for residents and tourists in order to obtain political influence (*i.e.* votes from residents) and welfare gains. Furthermore, coastal cities relying on income from tourists may control the number and location of firms near the beach to keep it clear and prevent excessive noise or pollution. The location of shopping centers close to ring roads around big cities such as London or Madrid is a strategic choice which may be influenced by zoning restrictions. Many large cities in the world such as México D.F, Paris, London or New Delhi are using zoning as a policy tool to regulate traffic (*i.e.* restriction of car access to city areas or any other type of zoning). These types of policies effectively crowd-out firms through regulation by forcing them to locate their activities in unrestricted areas. Zoning may, therefore, be a useful tool for urban design or industrial policies; nevertheless, it may have adverse effects when wrongly implemented. Thus, non-regulation and free entrance may be optimal in terms of welfare under certain assumptions.

However, when zoning is implemented authorities need to find regulating instruments (*i.e.* strategic location choices) which enable them to reduce harmful external factors efficiently. Authors like Mills (1989), Henderson (1991), Miceli (1992) and Wheaton (1993) have shown that zoning may become an interesting urban planning tool. This study focuses on the design of regulation in a duopolistic framework which includes a regulator with alternative political profiles.

Spatial competition models provide strong insights to study zoning policies. One of the most widely extended models in this field is Hoteling's (1929). There are two crucial assumptions in the original setup depicted in the model: i) firms compete in a duopolistic framework; ii) consumers are distributed uniformly along the market space. Both premises are too restrictive to explain certain equilibrium patterns –for instance: endogenous location of consumers–. Thus, Hotelling's model and some of its premises have been reconsidered to encompass a higher number of location patterns.

On one hand, when the model is expanded to an oligopolistic competition framework to allow entrance of more than one firm a strong version of the minimum differentiation principle holds. In fact, Economides (1993) finds that equilibrium in location does not exist in Hotelling's linear model when the number of firms is increased to three or more. The reason is that firms have a strong incentive to approach the center but no equilibrium exists where the whole market is served. When boundaries in the space location (*i.e.* linear case) exist, equilibrium ceases to exist. However, for the circular case equilibrium is reestablished as boundaries disappear under this spatial configuration.

Another strand of models is developed for the analysis of equilibrium outcomes under non-simultaneous location of firms. Neven (1987) considers a sequential entry of firms in a differentiated industry within a Hotelling model where firms choose location (product) with subsequent price equilibrium. His findings show that early entrants locate first, impeding later competitors to locate between them. Several sunk costs are identified corresponding to different natural market configurations so that equilibrium may be found for two or more firms. The sequential entry competition dynamic is extended by Gupta (1992) to a more general framework by assuming perfect foresight from firms. By modelling a Stackleberg game for the location of firms Anderson (1987) offers an alternative explanation: equilibrium occurs with the first firm locating in the center whilst keeping higher profits than the rest. This outcome is coherent with the entry-deterrent and monopoly solutions. Lastly, Pu-Yan Nie (2011, 2013 & 2014) and Yang & Nie (2015) consider the influence of other variables such as search costs to explain location patterns –*i.e.* industrial clusters– in low transportation cost frameworks.

A regulator may be introduced in spatial competition framework by specifying a third stage in the location-price game. Since authorities serve the interests of consumers and firms it is common to define a welfare function as the sum of surplus for both economic agents. However, we use a social welfare function formalized as a linear combination of the firm's profit and the utility for consumers[1] (Hamoudi & Risueño, 2012). This welfare function is interpreted in terms of the profile and interests of the public authority.

Research on spatial competition introduces the role of the regulator by focusing mainly on optimal firm location, regulated zone dimensions, pricing, land usage[2] and social welfare effects. For instance, Lai & Tsai (2004) examine Hotelling's linear city model with restrictions on the location of firms. They

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1 Such weighted approach was defined by Baron & Myerson (1982) whereas Armstrong *et al.* (1994) used it to describe a welfare function.

2 See Fujita & Thisse (1986).

show that maximum differentiation holds under Bertrand competition and social welfare is improved. Tsai *et al.* (2006) analyses how zoning affects the location of firms and the rents derived from land. Chen & Lai (2008) investigate the effects of symmetric zoning in the linear city, finding that firms locate in equilibrium at the extremes of the zoning area under Cournot competition. These authors conclude that introducing a regulated zoning area can be welfare improving. Matsumura & Matsushima (2012) study a duopoly model with restrictions on the location of firms. Their objective is to analyze the effects on consumer's welfare. The model shown is related to the issue of urban sprawl in order to determine the allowed dimension of economic activities. Hamoudi & Risueño (2012) consider the effect of zoning regulation in duopolistic circular model with Bertrand competition where consumers and firms are situated in different geographical zones within a city. They show that the optimal size of the shopping area depends on the regulator's political profiles. Furthermore, Hamoudi *et al.* (2015) complete the analysis of the location of firms by studying different specification for transportation costs to establish an equivalence result for convex and concave transport cost functions. The authors show that only a concave specification yields equilibrium.

The rest of this article is organized as follows: Section 2 develops the circular model while Section 3 focuses on the linear model. In both cases, the model is first depicted in order to determine price-location equilibrium and under optimal zoning policies. Each part concludes with specific remarks on the effects of regulation over competition.

2. The circular model

2.1. The model

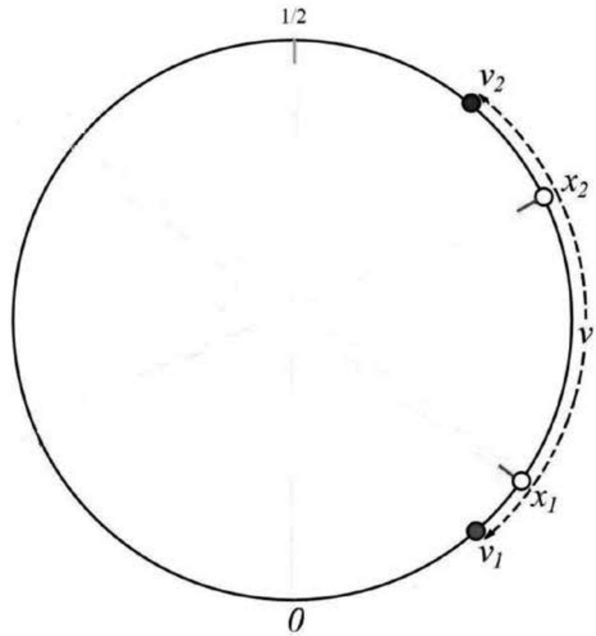
We study a location model where a regulatory authority is responsible for the design of an area within a unitary circular space. Any point in the perimeter of the circle corresponds to a number from the interval [0,1]. The southeasters' point is 0 and we move anti clockwise from there. Points 0 and 1 will therefore coincide. The planner divides the circle in two regions: the first one is a commercial area bounded by points v_1, v_2 such that $0 \le v_1 \le v_2 \le 1/2$ [3] where firms and families locate. The second one is the residential area where only families locate (see Figure 1). There are two firms located at x_1 and x_2 such that $x_1 \le x_2$ and $x_1, x_2 \in [v_1, v_2]$ selling the same good in the commercial area at prices p_1 and p_2 respectively.

A continuum of consumers spreads uniformly along the city. Each consumer buys one unit of good and pays the cost of transporting the good from the firm's location to her residence. The transportation cost incurred by the consumer is assumed to be a quadratic function of distance. Specifically, the function is taken as:

$$c(d_i(x)) = b\, d_i^2(x),\ b > 0, \quad i = 1,2\,,$$

[3] Here, it is not necessary to consider $0.5 \le v_1 \le v_2 \le 1$.

FIGURE 1
CIRCULAR MARKET



where the distance $d_i(x) = |x - x_i|$ between location consumer x and the location firm x_i, is defined as the shortest distance on the circle between the two points x, x_i.

Let s be the gross surplus for an arbitrary consumer, x. We assume s is large enough ($s >> 0$) to allow all consumers to buy. Utility for consumer x from buying the good from firm i is, therefore, given as: $u_i(x) = s - p_i - c(d_i(x))$. A consumer purchases the product from firm i when $u_i(x) < u_j(x), \quad i = 1,2, \quad i \neq j$.

The model is then formalized as a three-stage game (Fudenberg & Tirole, 1987). In the first stage, the regulator chooses the optimum size of the commercial area; in the second stage firms choose their locations simultaneously; in a third stage firms decide on their prices at the same time. The game is then solved by backward induction. Given the size of the commercial area, equilibrium outcomes are found to substitute in the profit function of firms.

2.2. Price and location equilibrium

The fact that the location space for firms is restricted does not alter the location of indifferent consumers[4]. Thus, the demand function remains the same as for the unrestricted space case, De Frutos *et al.* (1999). We can then proceed with

4 An indifferent consumer, α buys from firm 1 or 2 so that; $u_1(\alpha) = u_2(\alpha)$.

a change of variable: $z = x_2 - x_1$, where z represents the distance between both firms, that is, the difference between the chosen characteristics. The demand function is represented as follows:

$$D_1(p_1, p_2, x_1, x_2, v) = \begin{cases} 1 & p_1 - p_2 \le -bz(1-z) \\ \left[\dfrac{p_2 - p_1}{2bz(1-z)} + \dfrac{1}{2}\right] & -bz(1-z) \le p_1 - p_2 \le bz(1-z) \\ 0 & bz(1-z) \le p_1 - p_2 \end{cases}$$

In addition, demand for firm 2 is: $D_2 = 1 - D_1$.

Once consumer demands are found, the profit functions can be calculated using the following relation: $B_i(p_i, p_j) = p_i D_i(p_i, p_j)$, para $i = 1,2 \quad j = 1,2 \quad$ for $i \ne j$.

The existence of Nash equilibrium in prices is guaranteed for any size of the market v and any value of z, since profit functions are strictly concave (again as in De Frutos *et al.*, 1999). The solution corresponds to: $p_1^N(z) = p_2^N(z) = bz(1-z)$. Consequently, the demand and profit functions can be written as:

$$D_1^N = D_2^N = \frac{1}{2}, \quad B_1^N(z) = B_2^N(z) = \frac{1}{2} b\, z\, (1-z)$$

Proposition 1:

There is a unique Nash location equilibrium for any commercial area given by: $[v_1, v_2]$, $x_1^N = v_1, \quad x_2^N = v_2$.

Demonstration: (See Appendix). ■

The perfect equilibrium subgame expressions for prices demand and profits are:

$$p_i^N(v) = bv(1-v), \quad D_i^N = \frac{1}{2}, \quad B_i^N(v) = \frac{1}{2} bv(1-v), \quad i = 1,2, \quad v = v_2 - v_1$$

Remarks

In the circular model under zoning regulation, the location pattern satisfies the maximum differentiation principle: $x_1^N = v_1$, $x_2^N = v_2$. We observe that if $v_2 - v_1 = 1/2$, the location equilibrium remains the same as in the circular model without zoning (*i.e.* De Frutos *et al.*, 1999). Therefore, zoning half of the circular market does not affect the location strategies of firms. The intuition behind the maximum differentiation result is straightforward: firms locate at $x_1^N = v_1$, $x_2^N = v_2$ in order to avoid competition and reap some spatial monopolistic rents.

The demands are equal and independent of the size v. Thus, zoning does not affect the structure of demand in the location equilibrium. Prices and profits are also equal to each other and they are increasing in v.

$$\frac{\partial p_1^N}{\partial v} = \frac{\partial p_2^N}{\partial v} = b(1-2v) \geq 0, \quad \frac{\partial B_1^N}{\partial v} = \frac{\partial B_2^N}{\partial v} = \frac{1}{2} b(1-2v) \geq 0 \; for \; \forall \; v \in \left[0, \frac{1}{2}\right]$$

If the regulator chooses a large value for v, competition decreases. Therefore, if firms were able to decide on their location, they will always be interested in the commercial area to be as large as possible in which case, $v = 1/2$. On other hand, the regulator can force both firms to undergo more competition which could eventually yield to zero profits. Indeed, if v tends to zero, prices and benefits will also approach as both firms engage in Bertrand competition. Subsequently, zoning regulation can be seen as an industrial instrument to limit firms' monopoly power.

Given that the location pattern in this zoning model still satisfies the maximum differentiation principle, the purpose of a regulator is to find the dimensions for a commercial area and an exclusively residential area.

2.3. Optimal zoning

Town planners take their decisions based on the interests of firms and consumers and for this reason the objective function is usually defined as the sum of firm's profit and consumer's utility. In contrast to this type of model, at this point, we use an objective function for the regulator described as a linear combination from profits (firms) and utility (consumers). This, therefore, allows us to introduce the possibility to formally represent the preferences of a regulator in terms of the weight attached to surplus from producers and consumers. The welfare function can now be written as:

$$W(v) = \lambda B^N(v) + (1-\lambda) U(v).$$

Where:

- λ is the weight given by the regulator to firms. Thus, $(1 - \lambda)$ accounts for the weight given to consumers.
- $B^N(v) = B_1^N(v) + B_2^N(v)$ is firms' profit.
- $U(v) = S - \left[B^N(v) + C_T(v)\right]$, is the surplus from consumers.
- S is gross utility from consumers.
- $C_T(v)$ means total transport cost paid by consumers.

Total transport cost is then formalized as: $C_T(v) = I_1 + I_2$

- I_1 corresponds to total transport cost paid by consumers when they buy the good from seller 1.
- I_2 is the total transport cost paid by consumers when they buy the product from seller 2.

$$I_1 = \int_0^{\alpha_1^N} \left[b(x - x_1^N)^2 \right] dx + \int_{\alpha_2^N}^1 b\left[(1-x) + x_1^N \right]^2 dx; \quad I_2 = \int_{\alpha_1^N}^{\alpha_2^N} \left[b(x_2^N - x)^2 \right] dx$$

Where α_1^N, α_2^N are indifferent consumers in terms of buying from firm 1 or firm 2.[5]

We can then represent $C_T(v)$ as follows: $C_T(v) = \frac{b}{12}(3v^2 - 3v + 1)$.

- $C_T(v)$ decreases as the size of v increases. The mixed location area for both consumers and firms is: $\frac{\partial C_T}{\partial v} = \frac{b}{4}(2v-1) \leq 0$, $\forall v \in \left[0, 1/2\right]$. When the regulator seeks to minimize the total transport cost for consumers (*i.e.* their disutility), $\lambda = 1/2$ in the objective function. The optimal size for this case is $v = 1/2$.

Utility for the total of consumers is given by: $U(v) = S - \frac{b}{12}\left(-9v^2 + 9v + 1\right)$.

- $U(v)$ is decreasing and reaches a maximum for v equal zero. In contrast to firms, consumers are interested in a minimum size for the mixed consumers-firms area which then turns into a single point, thus, $v = 0$. The price of the good is equal to zero for this value and transport cost reaches a maximum. Consumers pay a high price for transport cost but are compensated by a zero cost for the good. The intuition for this result is illustrated by the fact that consumers travel massively (*i.e.* underplay direct transportation costs) when a free good is offered.

Given the expression for the welfare function *W (v)* the following can be highlighted:

- When $\lambda > 1/2$, the regulator favors firm's interest over consumer's.
- However, when $\lambda < 1/2$, the opposite happens.
- When $\lambda = 1/2$, we have a neutral case in which the same weight is given to both groups: consumers and firms. Furthermore, the welfare function for this case is depicted as:

$$W(v) = 1/2\,[S - C_T(v)]$$

The optimal strategy for the regulator is given by:

$$v^O = \underset{v}{Arg\,Max}\ W(v),$$
$$s.t.\ 0 \leq v \leq (1/2)$$

5 Indifferent consumers $\alpha_1^N = \frac{v_1 + v_2}{2}$, $\alpha_2^N = \frac{v_1 + v_2}{2} + \frac{1}{2}$.

By substituting the expression for $B^N(v)$ y $C_T(v)$ in the objective function we obtain the following: $W(v)=(2\lambda-1)\,bv\ (1-v)+(1-\lambda)\left[S-\frac{b}{12}(3v^2-3v+1)\right]$

Proposition 2:

For $0\leq v\leq\frac{1}{2}$, the optimal size for the mixed firms-consumers area is given as:

$$v_C^*=\begin{cases} v_{C1}^*=0, & \textit{if}\quad 0\leq\lambda\leq\frac{3}{7}\\ v_{C2}^*\in\left[0,\frac{1}{2}\right], & \textit{if}\quad \lambda=\frac{3}{7}\\ v_{C3}^*=\frac{1}{2}, & \textit{if}\quad \frac{3}{7}\leq\lambda\leq 1\end{cases}$$

Demonstration: (See Appendix). ■

Remarks

Note that zoning occurs in different ways depending on the bias of the regulator, thus, effectively changing the spatial distribution of firms:

- If $0\leq\lambda\leq\frac{3}{7}$, the regulator is consumer-biased. Agglomeration is then obtained, that is, the location space for firms is reduced to a single point, $v_{c1}^*=0$. In this case, competition among firms is very intense (Bertrand type) which implies the price of all products is close to zero. These factors clearly benefit consumers as they can live in a larger area enjoying more welfare. On the other hand, in terms of industrial policy, the regulator allows only one characteristic of the good to be produced despite the possibility to produce two characteristics. In this case, we obtain the following results:
  i) Total utility for consumers: $U(0)=S-b/12$,
  ii) The profit function of firms equals: $B(0)=0$,
  iii) The welfare function can be expressed as: $W(0)=(1-\lambda)\,(S-b/12)$.

- If $\lambda=3/7$, social welfare remains constant and independent from the size of the commercial area. The regulator has no a priori preference on the dimension of the commercial area.

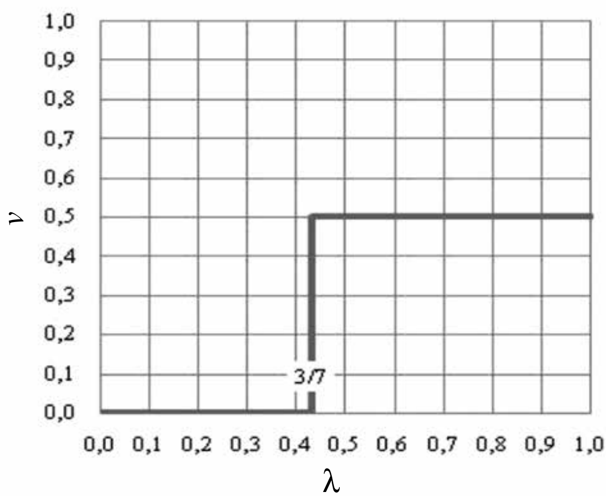
- When $\frac{3}{7}\leq\lambda\leq 1$, the regulator favors firms because the value of v, $v_{c3}^*=\frac{1}{2}$ is their preferred result. In this case, firms separate away from each other (*i.e.* dispersion), locating at the endpoints of the interval. $x_1=0,\quad x_2=\frac{1}{2}$. As a result, the regulator is supporting an industrial policy in favor of product variety. In this case:

i) Total utility for consumers: $U(1/2) = S - 13b/48$,
ii) Profit of firms: $B(1/2) = b/4$,
iii) The welfare function: $W(1/2) = (1-\lambda)\,S + (b/48)\,(25\lambda - 13)$.

We can observe that a firm biased regulator, $(3/7 \leq \lambda \leq 1)$, reduces total utility for consumers and improves profit for firms, $U(1/2) < U(0)$.

These results are represented in figure 3 by drawing the optimum size for the commercial area:

FIGURE 2
OPTIMUM SIZE FOR THE COMMERCIAL AREA



The *horizontal* axis shows the value of parameter λ whereas the vertical axis shows the value of parameter *v*. The thick line refers to the optimum size of the commercial area.

The optimal solution v^*, supports the popular view on the effects of the behavior of the authorities given their preferences.

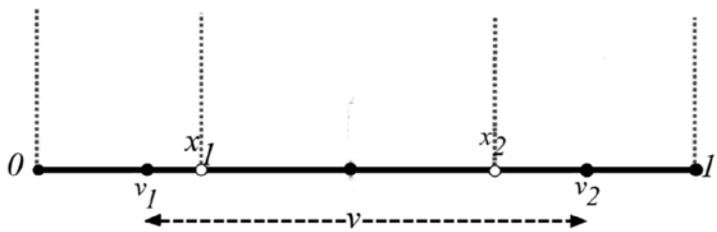
3. The linear city model

3.1. The model

As in the circular case, a regulator may restrain the production area to the segment (v_1, v_2). Inside this interval the location of firms is given by (x_1, x_2), so that $0 \leq v_1 \leq x_1 \leq x_2 \leq v_2 \leq 1$.[6] Consumers also distribute uniformly among the linear city of length one [0,1] where the areas $[0, v_1)$ y $(v_2, 1]$ are only residential. Again, this model is set up as a three stage game. In the first stage, a regulator chooses the size of the commercial area. In the second and third stages firms simultaneously decide on location and price.

We can represent the model as follows:

FIGURE 3
LINEAR MARKET



Using a quadratic function for transportation cost defined as:

$$c(d_i(x)) = b\, d_i^2(x),\ b > 0, \quad i = 1,2,$$

where the distance $d_i(x) = |x - x_i|$ between the location for consumer x and the location of firm x_i.

Thus, the indifferent consumer is: $\alpha = \frac{p_2 - p_1}{2b(x_2 - x_1)} + \frac{(x_2 + x_1)}{2}$.

Given the uniform distribution of consumers along the linear city, the results obtained for the regulated model are the same as in D'Aspremont *et al.* (1979) except for equilibrium in location given by: $x_1^N = v_1 ; x_2^N = v_2$. The following results are derived in this context:

$$p_1^N(v) = \frac{1}{3} b(v_2 - v_1)(2 + v_1 + v_2), \quad p_2^N(v) = \frac{1}{3} b(v_2 - v_1)(4 - v_1 - v_2)$$

$$\alpha^N(v) = \frac{1}{3} + \frac{(v_2 + v_1)}{6}$$

$$B_1(p_1, p_2) = \frac{b}{18}(v_2 - v_1)(2 + v_2 + v_1)^2, \quad B_2(p_1, p_2) = \frac{b}{18}(v_2 - v_1)(4 - v_2 - v_1)^2,$$

In order to compare profits between the firms $B_1^N(v) - B_2^n(v)$ is calculated.

6 Lai and Tsai (2004) comment on this type of zoning, nevertheless, they do not analyse it. In contrast, these authors, study the linear city model in [0, 1] by assuming the area [0, *z)* to be only residential.

$$B_1(p_1,p_2)-B_1(p_1,p_2)=\frac{2b}{3}(v_2-v_1)(1-(v_2+v_1))\geq 0 \quad if \quad v_1+v_2\leq 1$$

Given the equilibrium locations: $x_1^N = v_1 ; x_2^N = v_2$, the firm closer to the center of the commercial area obtains higher profits. Under symmetric zoning, $v_1 + v_2 = 1$ firms obtain the same profit. The regulator has no a priori preferences for any of the firms since both are private profit maximizing entities. In turn, we focused on the study of optimal zoning for the commercial area in the symmetric zoning case where: $v_1 + v_2 = 1$.

3.2. Optimal zoning

The last stage of the game for given prices and locations in equilibrium is then solved. The optimal size of the commercial area is determined by restricting the study to the symmetric case in which $v_1 + v_2 = 1$, where $v_2 - v_1 = v$.

Equilibrium locations under this condition are also symmetric to the extremes of the market. The above obtained results can then be expressed as:

$$x_1^N=\frac{1}{2}-\frac{v}{2}, \quad x_2^N=\frac{1}{2}+\frac{v}{2}, \ \alpha^N=\frac{1}{2}, p_1^N(v)=p_1^N(v)=bv, B_1^N(v)=B_2^N(v)=\frac{1}{2}bv.$$

Prices and profit, therefore, depend on the size of the area, v. Firms ideally prefer the maximum size for the commercial, $v = 1$.

As in the circular case, the objective function for the regulator is given as:

$$W(v)=\lambda B^N(v)+(1-\lambda)\left[S-B^N(v)-C_T(v)\right]$$

In this function λ, $B^N(v)$, S, $C_T(v)$, are respectively the weight given to firms by the regulator; the total profit, consumers' total surplus, and total transportation cost which is:

$$\begin{aligned} C_T(v) &= \int_0^{\alpha^N} b\left[x-1/2+v/2\right]^2 dx+\int_{\alpha^N}^1 b\left[x-1/2-v/2\right]^2 dx, \\ &= b(3v^2-3v+1)/12 \end{aligned}$$

Notice that the expression for transportation cost here is identical to the circular city case. In this case, however, $v \in [0,1]$ implies that:

– $C_T(v)$ is decreasing for $\forall\, v \in \left[0,1/2\right]$, is increasing $\forall\, v \in \left[1/2,1\right]$, and reaches its minimum value for $v = 1/2$, thus, firms locate in the following points: $x_1^N = 1/4, x_2^N = 3/4$, for which $C_T(1/2) = 1/48$. This is an identical result to the optimal social value stemming from the unrestricted linear city model seen before.

– The total price $\left[B^N(v)+C_T(v)\right]=b\left(3v^2+9v+1\right)/12$ paid by consumers reaches a minimum for v equal zero:
$\partial\left[B^N(v)+C_T(v)\right]/\partial v = b\,(2v+3)/4 \geq 0 \;\; for \;\; \forall v \in \left[0,1\right]$.

Due to the above argument, consumers are interested in the size of the commercial area being reduced to a single point, $v = 0$, which involves firms locating at exactly the same point: $x_1^N = x_2^N = 1/2$. The price of the good is equal to zero for this value but transport cost reaches a maximum. Therefore, it is worth for consumers pay high transport costs since that will mean a null price $p_1^N(0) = p_1^N(0) = 0$, for the good as in the circular case.

Given the expressions for $B^N(v) \; y \; C_T(v)$, objective function of the regulator is written as: $W(v) = (2\lambda - 1)\,bv + \dfrac{b}{12}(1-\lambda)\,(S - 3v^2 + 3v - 1)$.

Proposition 5:

For $0 \leq v \leq 1$, the optimal size of for the mixed consumers-firms area is given by:

$$v_L^* = \begin{cases} 0, & if \;\; 0 \leq \lambda \leq \dfrac{3}{7} \\ \dfrac{7\lambda - 3}{2(1-\lambda)}, & if \;\; \dfrac{3}{7} \leq \lambda \leq \dfrac{5}{9} \\ 1, & if \;\; \dfrac{5}{9} \leq \lambda \leq 1 \end{cases}$$

Demonstration: (See Appendix). ■

Remarks

– If the regulator is consumer-biased $(\lambda \leq 3/7 \approx 0{,}42)$ and the commercial area is restricted to $v_L^* = v_{L1}^* - v_{L2}^* = 0$, the endpoints of the restricted interval coincide with $v_{L1}^* = v_{L2}^* = 1/2$. Firms subsequently locate at the same point, $x_1^N = x_2^N = v_{L1}^* = v_{L2}^* = 1/2$. They engage in Bertrand competition, with null prices $(p_1^N(v) = p_2^N(v) = 0)$. Agglomeration takes place and minimum differentiation in product variety (industrial policy) holds. By calculating, and, the following results are obtained:
  i) Utility for the total of consumers is: $U(0) = S - b/12$,
  ii) The profit function for the firms equals: $B(0) = 0$,
  iii) The welfare function can be expressed as: $W(0) = (1-\lambda)\,(S - b/12)$

– If the regulator's bias (λ) takes some value between 3/7 and 5/9; the optimal size v_L^* reaches a minimum in the lower end of the interval for λ = 3/7, and a maximum in the upper end for λ = 5/9, that is, for $v_L^*(3/7) = 0$, $v_L^*(5/9) = 1$.

In this case $0 \le v_L^* \le 1$ is increasing, meaning that when λ is higher the value of v is larger. We then move from agglomeration to dispersion (from minimum differentiation to maximum differentiation). When planner has more incentives to overvalue consumer, he assigns a value of λ = 1/2, the optimal size of the commercial area is reached for $v_L^* = 1/2$. In this case, $U(0)$, $B(0)$ and $W(0)$ are given by:

i) Total utility for consumers is: $U(\frac{7\lambda-3}{2(1-\lambda)}) = S - \frac{b}{48(1-\lambda)^2}(25\lambda^2 + 46\lambda - 23)$,

ii) The profit function of firms is: $B(\frac{7\lambda-3}{2(1-\lambda)}) = b\frac{7\lambda-3}{2(1-\lambda)}$

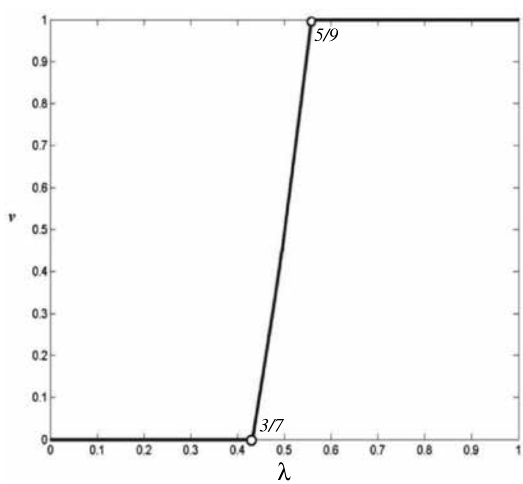
iii) The welfare function is: $W(\frac{7-\lambda}{2(1-\lambda)}) = (1-\lambda)\, S - \frac{b}{48(1-\lambda)}(143\lambda^2 - 118\lambda + 23)$.

– Finally, if the regulator is firms-biased, $\lambda \ge 5/9 \approx 0{,}55$, then $v_L^* = 1$, or identically, $x_1^N = v_{L1}^* = 0$, $x_2^N = v_{L2}^* = 1$. The mixed consumers-firms area corresponds to the interval [0, 1]; firms can locate in the whole market and they choose maximum differentiation in terms of product variety. This corresponds to agglomeration when interpreted in terms of location patterns. Now, $U(0)$, $B(0)$ and $W(0)$ correspond to:
i) Utility for the total of consumers: $U(1) = S - 13b/12$,
ii) The profit of firms is: $B(1) = b$
iii) The welfare function is: $W(1) = (1-\lambda)\, S + (b/12)\,(25\lambda - 13)$.

Similar to the circular model when the regulator is firm-biased and (3/7 ≤ λ ≤ 1 the total utility for consumers decreases and $U(1/2) < U(0)$. Firm's profits improve in this context: $B(1/2) > B(0)$.

We can represent these analytical results in the following figure:

FIGURE 4
LINEAR MODEL



Again, the optimal solution v^* in the linear model, supports the popular belief on the behavior of the authorities.

The results for the optimal size of the mixed commercial-residential are the same for the linear and the circular model, as shown in the following table:

Circular Market: $0 \leq v_C \leq 1/2$	Linear Market: $0 \leq v_L \leq 1$
Optimal Zoning $\begin{cases} \text{if} \quad 0 \leq \lambda \leq \frac{3}{7}, & v_C^* = 0, \\ \text{if} \quad \lambda = \frac{3}{7}, & v_C^* \in \left[0, \frac{1}{2}\right], \\ \text{if} \quad \frac{3}{7} \leq \lambda \leq 1, & v_C^* = \frac{1}{2}, \end{cases}$	Optimal Zoning $\begin{cases} \text{if} \quad 0 \leq \lambda \leq \frac{3}{7}, & v_L^* = 0, \\ \text{if} \quad \frac{3}{7} \leq \lambda \leq \frac{5}{9}, & v_L^* = \frac{7\lambda - 3}{2(1-\lambda)}, \\ \text{if} \quad \frac{5}{9} \leq \lambda \leq 1, & v_L^* = 1, \end{cases}$

4. Conclusions

In this article we analyse spatial competition in a regulated market where consumers locate freely along the market space, whereas firms are obliged to locate in a restricted area. We study the influence of regulation on competition in a circular space and then extend the analysis to a linear space. This article contributes to the spatial competition literature by providing a framework to investigate the role the regulator plays on the location pattern of firms. A specific functional form to model the behavior of the regulator is interpreted in terms of the values of the parameters. Equilibrium results are then shown to highlight similarities in the behavior pattern of firms, regardless of the specific spatial configuration. Interestingly, a concave specification for transportation cost yields equilibrium outcomes.

This type of approach is fruitful from two perspectives. On one hand, equilibrium for both different spatial configurations can be categorized in terms of dispersion/agglomeration. This interpretation offers insights for urban policy. Given the weights the regulator places on consumers or firms, different location patterns will emerge from the location and price behavior of firms. Zoning may, therefore, become a useful tool from the perspective of urban policies and city planning. This is particularly relevant given the increasing role of cities –*i.e.* smart cities– in the economies of emerging and advanced countries.

On the other hand, these results can also be scrutinized from an industrial policy perspective as minimum or maximum differentiation cases. A regulator may, under these premises, exert influence on the location of firms to supply key markets. Zoning may turn into a useful tool for industrial policies. The political-economy issue of re-industrialization may be analyzed in light of the regulator bias towards firms of consumers. As a consequence, the results obtained for both spatial configurations can be interpreted in terms of the redistribution of welfare. It is proved that a regulator influences location patterns of firms, affecting the structure of the market. In this respect it can be stated that strong competition is triggered in the consumer-biased regulator case whereas weak competition arises in the case of a firms-biased regulator. For a neutral regulator we find moderate competition. These results contribute to establish theoretical support to understand the role of regulation in spatial competition.

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Appendix

Demonstration of Proposition 1:

Given the expressions for the price equilibrium profit functions; the Nash equilibrium locations can be calculated by using the first order condition: $\frac{\partial B_1}{\partial x_1}=0,\ \frac{\partial \cancel{B}_2}{\partial x_2}=0,$ We obtain that $x_2 - x_1 = \frac{1}{2}$

Since $0 \le v_1 \le x_1 \le x_2 \le v_2 \le 1/2,$ for any value of x_1, x_2, we then have that $x_2 - x_1 \le 1/2$, so that the necessary condition $\frac{\partial B_1}{\partial x_1}=0,\ \frac{\partial \cancel{B}_2}{\partial x_2}=0,$ is only fulfilled for locations: $x_1^N = v_1,\ x_2^N = v_2$. ■

Demonstration of Proposition 2:

For clarity reasons, the welfare function is rewritten as follows:

$$W(v)=\frac{b}{4}(7\lambda-3)\, v\, (1-v)+(1-\lambda)(S-\frac{b}{12})$$

By using the first order condition we find that: $\frac{\partial W}{\partial v}=\frac{b}{4}(1-2v)(7\lambda-3)$. Considering that this condition depends on the value of parameter λ, in order to determine the maximum a second order condition is needed: $\frac{\partial^2 W}{\partial v^2}$.

Taking into account that: $\frac{\partial^2 W}{\partial v^2}=-\frac{b}{2}(7\lambda-3)=0$, we can deduct the following results:

$-If \quad \lambda \ge \frac{3}{7} \quad \Rightarrow \frac{\partial^2 W}{\partial v^2} \le 0$ the social welfare function is concave and reaches a maximum for $v = v_{c3}^* = \frac{1}{2}$

$-If \quad \lambda = \frac{3}{7} \quad \Rightarrow \frac{\partial^2 W}{\partial v^2} = 0$ the social welfare is constant which means it takes the same value for any value of v between 0 y ½. This means the maximum is reached for $v = v_{c2}^*$ so that $0 \le v_{c2}^* \le \frac{1}{2}$.

$-If \quad \lambda \le \frac{3}{7} \quad \Rightarrow \frac{\partial^2 W}{\partial v^2} \ge 0$, in this case the social welfare function is convex and the solution for the first order condition corresponds to a minimum, so that a maximum is obtained for $v = v_{c1}^* = 0$. ■

Demonstration of Proposition 3:

By using the first order condition, $\frac{\partial W}{\partial v} = \frac{b}{4}\left[(7\lambda - 3) - 2v(1-\lambda))\right] = 0$ and for $\lambda \neq 1$, it is found that $v_L^* = \frac{7\lambda - 3}{2(1-\lambda)}$. Assuming that the second order condition is: $\frac{\partial^2 W}{\partial v^2} = -\frac{b}{2}(1-\lambda) < 0$. Given that: $0 \leq v_L^* \leq 1$, v_L^*, is the maximum of the objective function if: $\frac{3}{7} \leq \lambda \leq \frac{5}{9}$.

–If $\lambda \leq \frac{3}{7} \Rightarrow v_L^* \leq 0$ *and* $\frac{\partial W}{\partial v} \leq 0$ *for* $\forall v \in [0,1]$, a maximum is reached for $v_L^* = 0$.

–If $\lambda \geq \frac{5}{9} \Rightarrow v_L^* \geq 1$ and $\frac{\partial W}{\partial v} \geq 0$ for $\forall v \in [0,1]$, the maximum is reached for $v_L^* = 1$. ■