On the Efficiency and Stability of a Two-Way Flow Network with Small Decay: A Note*

Sobre la eficiencia y la estabilidad en una red de flujo bidireccional con pequeño decaimiento: una nota

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Abstract

The seminal two-way flow strategic network formation model of Bala and Goyal (2000b) exhibits a substantial tension between stability and efficiency. In this note, I show that despite this tension every link receiver in a Nash network serves as an efficient transmitter of information assuming a small degree of information decay. Thus, a strategic decision of every link sender, who always forms links with efficient link receivers for his own interest, does in part lead to a socially desirable outcome. I also show how this finding can potentially refine some results in the literature.

Key words: *Network Formation, Nash Network, Two-way Flow Network, Agent Heterogeneity, Efficient Network.*

JEL Classification: *C72, D85.*

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Resumen

El modelo seminal de formación de redes estratégicas de flujo bidireccional de Bala y Goyal (2000b) presenta una tensión considerable entre la estabilidad y la eficiencia. En esta nota, muestro que, a pesar de dicha tensión, cada receptor de enlace en una red de Nash actúa como un transmisor eficiente de información si se asume un pequeño grado de decaimiento de la información. De este modo, la decisión estratégica de cada emisor de enlace, quien siempre forma enlaces con receptores de enlace eficientes en su propio beneficio, conduce en parte a un resultado socialmente deseable. Además, muestro cómo este hallazgo puede refinar algunos resultados existentes en la literatura.

Palabras clave: *Formación de redes, red de Nash, Red bidireccional, Agentes heterogéneos, Red eficiente.*

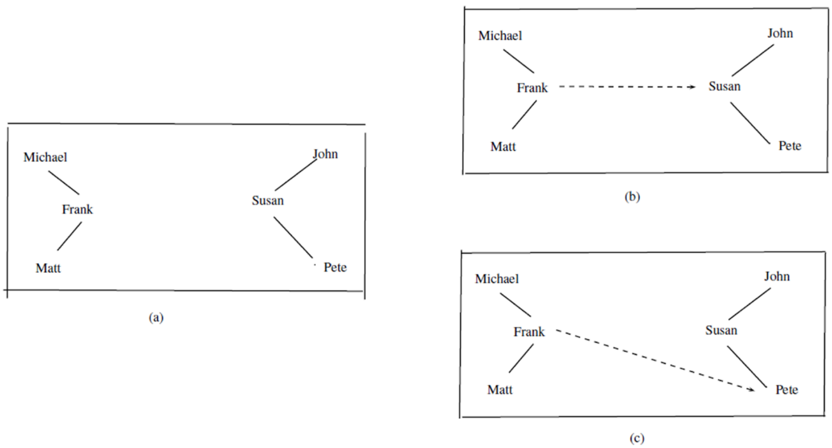
Clasificación JEL: *C72, D85.*

1. Introduction

In a social network, information transmission tends to be imperfect. In the literature of strategic network formation, a rigorously studied form of imperfect information transmission is the so-called 'small decay'. It captures the idea that the worth of information decays as it traverses through each link. Yet this decay is sufficiently small that, if a chain between two agents exists, then neither agent finds an incentive to shorten the distance through a costly link formation. This assumption of 'small decay' is rigorously studied in the context of two-way flow model of network formation with nonrival information by De Jaegher and Kamphorst (2015), which is an extended model of network formation proposed in the seminal paper of Bala and Goyal (2000b). Their major novel findings, which allow them to finely characterize the equilibrium networks, are; (i) best-informed agents, defined as agents who received more information than others, are attractive as link receivers and (ii) these best-informed agents are located 'in the middle' of other agents.

In this note, I complement these novel findings by showing that every best-informed agent, in addition to being an optimal choice as a link receiver from a strategic perspective of self-interest agents, is also an agent that transmits information *most efficiently* in the network. That is, within a group of agents M if a link sender i's best response is to choose a best-informed agent $j \in M$ as a link receiver, then j is also the agent that maximizes the total information that flows within the network. These results are established as Proposition 1 and Remark 2 in this note. See Figure Figure 1 in the Appendix for an intuitive, informal example of this insight[1].

FIGURE 1
EXAMPLE 1



Notes: On the left (picture (a)), two groups of agents are disconnected. If the decay is small, then within the group of Susan, Pete and John it can be shown that Susan, who is in the middle, possesses more informational quantity than Pete and John do (see lemma 1 in de jaegher and kamphorst (2015)). Consequently, on one hand, if Frank wants to acquire information with the group of Susan, John and Pete, then for the benefit of his own interest Frank will choose Susan as his partner, as shown on the upper right (picture (b)), rather than John or Pete, as shown on the lower right (picture (c)). On the other hand, proposition 1 in this note further shows that Susan - rather than John and Pete - is also an agent that efficiently transmits information in the network.

I provide a brief literature review and threefold contributions of this note as follows. First, existing literature in the strand of two-way flow network formation shows that there is a substantial tension between efficient network and equilibrium networks. Specifically, within the context of the aforementioned two-way flow model of network formation with nonrival information Proposition 1 and 2 in De Jaegher and Kamphorst (2015) show that the set of Nash networks can be large and does contain networks that have long diameters,

[1] This Figure is inspired by Figure 1 in De Jaegher and Kamphorst (2015).

while Proposition 5.5 in Bala and Goyal (2000b) shows that a star - a network with diameter of only 2 - is a unique minimal nonempty efficient network. Thus, considering the differences in terms of network architectures and diameters the tension between efficient networks and Nash networks is quite extremE[2 3]. However, despite this tension between stability and efficiency to my knowledge this note is the first work in the literature that provides a connection between efficient network and Nash network by precisely pinpoints the source of inefficiency in a Nash network; identities of link senders.

Second, my result shows that in a Nash network a strategic choice made by self-interested agents who act as link senders, which results in every link receiver being efficient, does in part lead to a socially desirable outcome. Put differently, the fact that each link sender has to bear the entire link formation cost does not cause a link sender to choose an inefficient link receiver. Thus, by assuming a less general form of payoff function my result stands in contrast to the remark of Jackson (2008), "there are still inefficiencies, most notably since only one player bears the cost of a link while many players benefit from its existence."

Third, the above result also becomes a technical tool that refines some existing results in the literature. By incorporating small decay assumption, multiple efficient networks (assuming no decay) can be ranked in terms of efficiency of information flow. Consequently, a large set of efficient networks - which is a technical difficulty in some existing work - can be substantially narrowed down. At the same time, the incorporation of small decay assumption helps understand which efficient networks under the assumption of perfect communication are likely to be resilient to a small degree of imperfect communication. In Section 4, I show how to use this technical tool to refine the results of Unlu (2018), which finds that agent heterogeneity can lead to a large set of efficient networks that contains long diameter networks.

2 A similar tension also exists if no decay and agent homogeneity or agent heterogeneity as in Unlu (2018) is assumed. For more details, Unlu (2018) elaborates on this conflict found in Jackson and Wolinsky (1996), Bala and Goyal (2000b) and Bala and Goyal (2000a).

3 Indeed, such a tension has also been mentioned by Breitmoser and Vorjohann (2013). In par ticular, Breitmoser and Vorjohann (2013) remark that stars or complete networks are efficient across various models of network formation, including two-way flow model with bilateral link consent by Jackson and Wolinsky (1996), two-way flow model with cut-off decay (Hojman and Szeidl (2008)), model with far-sighted players (Dutta et al. (2005)), model with endogenous link strength (Bloch and Dutta (2009)) and model with transfer payments between players (Bloch and Jackson (2007)). Breitmoser and Vorjohann (2013) complement the literature by showing that substantially differ ent architectures of networks- redundant, incomplete and circular networks- are efficient if noisy communication is assumed. See the first two paragraphs in Breitmoser and Vorjohann (2013) for a comprehensive literature review.

This note proceeds as follows. Section 2 introduces the model and two important concepts - efficient link receiver/transmitter and best-informed agent. Section 3 provides main results. Section 4 provides applications of the main results to the literature. Section 5 concludes with remarks on further potential studies.

2. The Model

This note primarily follows the notation of De Jaegher and Kamphorst (2015), since it is the paper that this note seeks to complement.

2.1 Link Establishment And Individual's Strategy

Let $N = \{1,...,n\}$ be the set of all agents. An agent $i \in N$ can form a link with another agent j without j's consent. ij denotes such a link. The set of all possible links that i forms is $L_i = \{ij; j \in N \setminus \{i\}\}$. The set of all possible links is $L \equiv \cup_{i\in N} L_i$. $g_i \subset L_i$ is a *strategy* of i and $g = \cup_{i\in N} g_i$ is a *strategy profile*. A strategy space G is $G \equiv 2^L$. Pictorially, a strategy profile g is also a network, where an arrow from agent i to j indicates that $ij \in g_i$.

2.2 Information Flow

Information flow is two-way in the sense that it flows between two agents regardless of who sponsors the link, hence the term 'two-way flow model'. Let $\overline{ij} \in g$ represents that either $ij \in g$ or $ji \in g$. Information can also flow via a *chain*. A chain between i and j in a network g, denoted by $P_{ij}(g)$, is a sequence of agents $\{\overline{i_0 i_1},...,\overline{i_{k-1} i_k}\}$ such that $i_0 = i, i_k = j$. If there is a chain between i and j, we say that i and j are connected. A *shortest chain* between i and j is, of course, the chain(s) between i and j with the least amount of links. The distance between i and j, denoted by $d_{ij}(g)$ is defined as the amount of links of the shortest chain(s). If $j = i$ then we assume, following the literature, that the distance between i and himself is 0. If j and i are not connected, then we set $d_{ij}(g) = \infty$.

2.3 Information Decay

Let $\sigma \in [0,1]$ denote the *decay factor*. For example, if the value of information that an agent j possesses is 1 and the distance between i and j is k then the information that i receives from j in g is σ^k.

2.4 Small Decay Assumption

Suppose that information of j flows to i via a multi-link chain, then i can improve the information flow by establishing a link that results in a shorter chain. Such an incentive arises if the improvement in terms of information flow exceeds the increasing link establishment cost. However, if the decay factor σ is close to 1 then the improvement in terms of information flow becomes marginal and, consequently, such an incentive to establish a link diminishes. By the same analogy, from an efficiency perspective the benefits to all agents in the network relative to the cost of establishing an extra chain also diminishes if the decay is sufficiently small. As a result, there is at most only one chain between any pair of agents. This small decay assumption is assumed in De Jaegher and Kamphorst (2015) and will be assumed throughout this note. See Lemma 1 that precedes Proposition 1 in the next section.

2.5 Network-Related Notations

A subnetwork of g is a network g' such that $g' \subset g$. A network is said to be *connected* if there is a chain between every pair of agents in the network. g', a subnetwork of g, is said to be a component of g if g' is a maximal connected subnetwork of g. A network is *empty* if no agent forms a link. A non-empty component of a network or a network is *minimal* if there is at most one chain between any pair of agents in the network. An agent i is called a *link sender* (*receiver*) if there is a link $xy \in g$ such that $x = i$ ($y = i$). A minimally connected network is a *star* if there is an agent i such that $ij \in g$ for every $j \neq i >$ but $jk \notin g$ for every $j, k \neq i$.

Next, I introduce some notations concerning information flow. Let g be a minimally connected network. Due to the fact that there is only one chain between every pair of agents in g, a removal of the link $\overline{ij} \in g$ further splits g into two disconnected subnetworks - one containing i and the other one containing j. Let $D^i_{\overline{ij}}(g)$ and $D^j_{\overline{ij}}(g)$ denote these two subnetworks respectively. Furthermore, let $N^i_{\overline{ij}}(g)$ and $N^j_{\overline{ij}}(g)$ be the sets of agents in these two networks respectively. These notations will later be used to establish the concept of efficient link receiver.

2.6 Modified Networks

$g - ij$ is defined as $g - ij = g \setminus \{ij\}$. That is, $g - ij$ is modified from g by simply removing the link $ij \in g$. Similarly, $g + ij = g \cup \{ij\}$ is the network g modified by adding the link ij. Of course, $g - ij + kl = (g \setminus \{ij\}) \cup \{kl\}$ is the network g modified by removing the link ij and adding the link kl.

Next, consider two disconnected networks g' and g'' and assume that agents i and j are in g' and g'' respectively, then we define $g' \oplus_{ij} g''$ as the network that results from joining the two networks g' and g'' through the addition of the link ij. That is, $g' \oplus_{ij} g'' = g' \cup g'' \cup \{ij\}$. Note that if $ij \in g$ then $D_{ij}^{i}(g) \oplus_{ij} D_{ij}^{j}(g) = g$.

2.7 Quantity Of Information

Let the value of information that each agent possesses be 1. Let $I_{ij}(g) = \sigma^{d_{ij}(g)}$. That is, I_{ij} is the quantity of information that i receives from j in g. We then define the total information that i receives from every agent in the network as $I_i(g) = \sum_{j \in N} I_{ij}(g) = \sum_{j \in N} \sigma^{d_{ij}(g)}$. Observe that $I_{ii}(g) = 1$.

2.8 Cost Function And The Payoffs

Let $N_i^S(g) \equiv \{j \in N; ij \in g\}$ denote the set of all agents with whom i establishes a link. In most of the literature in the strand of two-way flow model including the pioneering work of Bala and Goyal (2000b) (Proposition 5.5) and recent works of Olaizola and Valenciano (2021) and Hoyer and Jaegher (2023), cost function is assumed to be linear, agent homogeneity in link formation cost is also assumed and the benefit that each agent receives is precisely the total information that he receives. To allow my results to be compared with those of Bala and Goyal (2000b), this note will also adopt these assumptions. This leads to the following payoff;

$$U_i(g) = I_i(g) - n_i^S(g)c = \sum_{j \in N} \sigma^{d_{ij}(g)} - n_i^S(g)c$$

where $n_i^S(g) = \left|N_i^S(g)\right|$.

2.9 Nash Networks

Consider a network g^* such that a strategy of i is $g_i^* \subset g^*$ >. Let $g_{-i}^* = g^* \setminus g_i^*$ so that $g^* = g_i^* \cup g_{-i}^*$. g_i^* is said to be a *best response* of i if $U_i(g^*) \geq U_i(g_i \cup g_{-i}^*)$ for every g_i which is a strategy of i. g^* is said to be a *Nash network* if every agent chooses his best response.

2.10 Efficiency Of A Network

Let $W(g) = \sum_{i=1}^{n} U_i(g)$. A network g *dominates* another network g' if $W(g) \geq W(g')$. A network g is efficient if it dominates every other network. Consider the payoff as in Eq. 1, we can express $W(g)$ as

$W(g) = \sum_{i \in N} I_i(g) - \sum_{i \in N} n_i^S(g)c$. We denote the first term on the right as $\bar{I}(g) = \sum_{i \in N} I_i(g)$ and call it total informational quantity of the network g. Similarly, We denote the second term on the right as $\bar{C}(g) = \sum_{i \in N} n_i^S(g)c$ and call it total cost of the network g.

2.11 Efficient Link Receiver

In a minimally connected network g, consider a link $\overline{xy} \in g$. j' *is superior to* j'' *as a transmitter* with respect to the link $\overline{xy}$ if $j', j'' \in N^y_{\overline{xy}}(g)$ and $\bar{I}\left(g - \overline{xy} + \overline{xj'}\right) \geq \bar{I}\left(g - \overline{xy} + \overline{xj''}\right)$. Moreover, j' is said to be an *efficient transmitter* with respect to the link $\overline{xy}$ if j' is superior to every agent in $N^y_{\overline{xy}}(g)$ as a transmitter. A link receiver j is said to be an *efficient link receiver* if j is an efficient transmitter with respect to every link $xy \in g$ such that $y = j$. In a network g, a link sender i is said to be an *efficient link sender* if i is an efficient transmitter with respect to every link $xy \in g$ such that $x = i$.

Remark 1. *Assuming the payoff as in Eq. 1, the following can be said about a minimal efficient network;*

1. *A minimal efficient network is such that every link receiver and every link sender is an efficient transmitter. That is, every agent in the network is an efficient transmitter.*
2. *Since a star is the unique efficient network within the class of non-empty minimal network*[4]*, a star is a unique network such that every link sender and every link receiver is efficient.*

2.12 Best Informed Agent

In a minimally connected network g, Let $M \subset N$ be a minimally connected subset of agents and $i, j \in M$. i is *better informed* than j in the set M if $\sum_{k \in M} I_{ik}(g) \geq \sum_{k \in M} I_{jk}(g)$. If i is better informed than every other agent in the set M, then i is said to be a best-informed agent in the set M. Alternatively, if $M = N^x_{\overline{xy}}(g)$ for a link $\overline{xy} \in g$, we then say that i is better informed than j with respect to the link $\overline{xy}$ and i is *best-informed* with respect to the link $\overline{xy}$.

Observe the following differences between the definitions of best-informed agent and efficient transmitter. The definition of best-informed agent revolves around the informational quantity *received by each individual*, while the definition of efficient transmitter revolves around the total informational quantity *received by all agents* in the network. Despite these differences, it turns out

4 *The result that a star is the unique architecture of nonempty minimal efficient network is stated in Proposition 5.5 of Bala and Goyal (2000b).*

that the identity of an efficient transmitter and best-informed agent is identical. This is proven in the next section.

3. Main Analysis: Proposition 1

I first state a lemma that establishes the threshold level of 'small decay' that guarantees that both efficient network and Nash networks are minimally connected. I then relate the concept of efficient transmitter and the concept of best informed agent in Proposition 1. All proofs are relegated to the Appendix.

Lemma 1. *Let the payoff be as in Equation 1. For any $c > 0$ and $n \geq 4$ there exists a threshold level of decay $\sigma_K < 1$ such that for all $\sigma > \sigma_K$ every nonempty efficient network and every nonempty Nash network is minimally connected.*

Importantly, this Lemma 1 implies that the small decay assumption as used in this note is slightly different from that of De Jaegher and Kamphorst (2015). σ_K above guarantees that all nonempty efficient networks and Nash networks are minimally connected, while the threshold level of decay σ_M as in Lemma 4 of De Jaegher and Kamphorst (2015) only guarantees that all nonempty Nash networks are minimally connected. Such a threshold is necessary since Remark 2 in this note intends to establish a connection between nonempty efficient network and nonempty Nash network, while De Jaegher and Kamphorst (2015) intend to provide the characterization of nonempty Nash networks that is minimally connected[5].

Next, a pivotal lemma that is the basis for Proposition 1 is established below.

Lemma 2. *Let g' and g'' be two minimally connected networks such that $g' \cap g'' = \varnothing$ and N' and N'' be the set of agents in g' and g''*

[5] Note that the result of Proposition 1 below would still hold even if σ_K is replaced by σ_M as in Lemma 4 of De Jaegher and Kamphorst (2015) since Proposition 1 only points out that every link receiver in a minimally connected Nash network is efficient. Note also that σ_K above is never less than σ_M for any cost c. Intuitively, the existence of σ_M as in De Jaegher and Kamphorst (2015) rests upon the fact that the benefit for a link sender from establish a nonminimal link becomes marginal once the decay is sufficiently small and hence cannot outweigh the additional link formation cost. σ_K above also rests upon the same analogy except that the decay also needs to be sufficiently small to guarantee that the benefit to *all agents* cannot outweigh the additional link formation cost. Finally, note also that because $\sigma_K \geq \sigma_M$ Proposition 1 below is established having in mind that all properties of nonempty minimal Nash networks are as characterized in Proposition 1 of De Jaegher and Kamphorst (2015).

respectively. Let $x \in N'$ *and* $y \in N''$. *Define* $g = g' \oplus_{\overline{xy}} g''$. *Then,*

1. $\sum_{i \in N'} I_i(g) = \overline{I}(g') + \sigma I_x(g') I_y(g'')$
2. $\sum_{i \in N''} I_i(g) = \overline{I}(g'') + \sigma I_x(g') I_y(g'')$
3. *and hence, as a corollary,* $\overline{I}(g) = \overline{I}(g') + \overline{I}(g'') + 2\sigma I_x(g') I_y(g'')$

This Lemma states that the total information in a network - $\overline{I}(g)$ - can be calculated by assuming as if the the network is split via a link $xy \in g$. $\overline{I}(g)$ then depends $\overline{I}(g')$, $\overline{I}(g'')$, $I_x(g')$ and $I_y(g'')$. Thus, if a link $xy \in g$ is replaced by another link xz where z also belongs to the same component as y, then whether total information in the network would improve depends solely on whether $I_z(g'') > I_y(g'')$. This leads to the main result of this note below:

Proposition 1. *Let the payoff be as in Equation 1,* $n \geq 4$ *and let the decay be* $\sigma \in (\sigma_K, 1)$. *In a minimally connected network consider a link* $\overline{xy} \in g$, j' *is superior to* j'' *as a transmitter with respect to the link* $\overline{xy}$ *if and only if* j' *is better informed than* j''. *Consequently, (i)* j' *is an efficient transmitter if and only if* j' *is best informed with respect to the link* $\overline{xy}$, *(ii) in a Nash network every link receiver is an efficient link receiver and (iii) A minimally connected Nash network and a minimally connected efficient network share a similarity: every link receiver is an efficient link receiver*[6].

The proof is given in the Appendix and the intuition is given as follows. Lemma 1 in De Jaegher and Kamphorst (2015) shows that an agent whose network position is 'in the middle' of the other agents tends to be a best informed agent because his position implies that each chain through which information *arrives to him* is relatively short, resulting in him suffering less decay[7]. By the same analogy, being in the middle means that each path through which information *arrives to other agents* from him is also relatively short, resulting in the fact that information that reaches to other agents suffers relatively less decay. In other words, once an agent's position is optimal for receiving information

6 *A straightforward corollary of this Proposition 1 is that in a minimally connected efficient network every link sender and link sender are efficient and, equivalently, best-informed.*

7 Definition 2 in De Jaegher and Kamphorst (2015) defines a middle agent as follows. "Consider a minimal connected subset of players M, $M \subset N$. We say that player j is in the middle of set M in network g if for each neighbor k of j in network g the following holds: in g more than half of the players in M (including k and j) are closer to j than to k." Lemma 1 in De Jaegher and Kamphorst (2015) then guarantees that a middle agent is always a best-informed agent in a network. More generally, even if there is no agent whose position is in the middle, Lemma 1 in Charoensook (2020) shows that a "positionally optimal" agent always exists.

for his own benefit, it also becomes optimal for transmitting information for the benefits of others. Example 1 below illustrate this intuition.

Example 1. *Let us reconsider our motivating illustration mentioned in the Introduction as in Figure 1. In Figure 1(a), we can see that Susan's position is in the middle of John and Pete. This makes her a unique best-informed agent among the three. Indeed, if we set* $\sigma = 0.99$ *then* $\bar{I}(Susan)1+2x0.99=2.98 > \bar{I}(John)=\bar{I}(Pete)=2.97=1+0.99+0.99^2$. *Moreover, according to our Proposition 1 Susan's position also makes her a superior transmitter of information relative to John and Pete. To understand why, observe that if John establishes a link with Susan as in Figure 1(b), this link yields him the benefits of* 2.98σ *due to the fact that he becomes one link away from Susan. By the same analogy, Matt and Michael receive the benefits of* $2.98\sigma^2$ *due to the fact that they are two links away from Susan.*

On the other hand, if Frank establishes a link with John or Pete, who is less informed than Susan, as in Figure 1(c)? Then from this link establishment Frank receives 2.97σ, *Mike and Matt receive* $2.97\sigma^2$. *Because* $2.97\sigma < 2.98\sigma$ *and* $2.97\sigma^2 < 2.98\sigma^2$, *we conclude that Frank, John and Pete always receive less informational quantity if Frank establishes a link with John or Pete rather than with Susan. Thus, Susan is an efficient transmitter and, according to the previous paragraph, a better informed agent relative to John and Pete. This echoes what our Proposition 1 states "* j' *is superior to* j'' *as a transmitter with respect to the link* $\overline{xy}$ *if and only if* j' *is better informed than* j''*."*

In terms of strategic action, observe also that Frank would also prefer to form a link with Susan rather than John or Pete because he always receives less if the link with Susan is replaced by a link with John or Pete ($2.97\sigma < 2.98\sigma$*). Thus, Susan is an optimal choice as a link receiver from the strategic point of view of Frank and also, according to the previous paragraph, an efficient transmitter. This echoes what our Proposition 1 states, "...in a Nash network every link receiver is an efficient link receiver."*

Remark 2. *Let the payoff be as in Equation 1,* $n \geq 4$ *and let the decay be* $\sigma \in (\sigma_K, 1)$, *a minimally connected efficient network, which is a star, and a Nash network have the following difference*[8]. *In a minimally connected Nash network a link sender is not necessarily efficient, while it is so in a minimally connected efficient network.*

8 *The result that a star is a unique architecture of nonempty minimally connected efficient network is stated in Proposition 5.5 of Bala and Goyal (2000b).*

4. Applications To The Literature

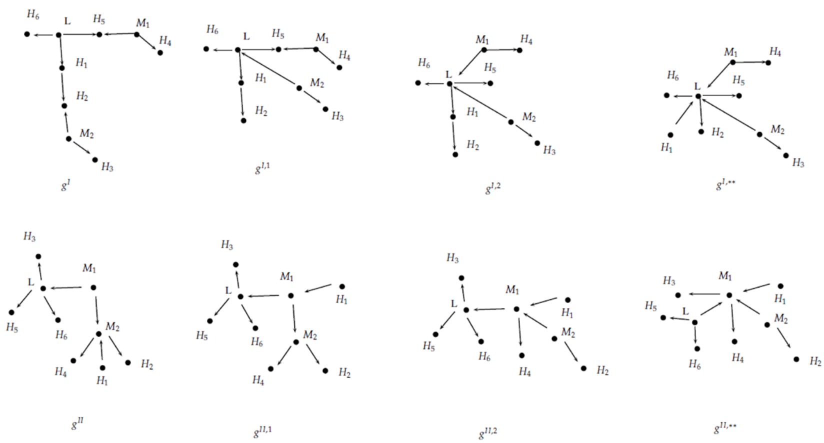
Finally, I turn to discuss some potential applications of Proposition 1 to the literature. Specifically, a major technical difficulty in the literature is that without decay the set of efficient networks can be very large. This section develops two algorithms that can, in part, solve this problem. Example 2below provides a motivational case in point.

Example 2. *Consider the following example, which is modified from Example 1 in Unlu (2018). Let* $n=9$*. Let* c_{ij} *be the link formation cost that* i *pays to establish a link with* j*. Let us assume player cost heterogeneity. That is,* $c_{ij}=c_i$ *for every* $i\neq j$*. Let* $\sigma=1$ *(i.e., no decay),* $V=100$ *where* V *is the value of information possessed by each agent and let the payoff be* $\pi_i(g)=100\sum_{j\in N}\sigma^{d_{ij}(g)}-(n_i^s c_i)^2$[9]*. Let agents consist of 3 groups:* $\{L\}$, $\{M_1,M_2\}$, $\{H_1,H_2,H_3,H_4,H_5,H_6\}$ *and set* $c_L=1.2$, $c_{M_1}=c_{M_2}=1.5$, $c_{H_1}=c_{H_2}=c_{H_3}=c_{H_4}=c_{H_5}=3$*. As in Unlu (2018) without decay, all networks in Figure 2 are efficient. To see why, observe that all these networks have the same total informational quantity (due to no decay assumption) and total link formation cost since agent* L *sponsors one link, intermediate cost agents* M_1,M_2 *sponsor two links and high cost agents sponsor no links except* H_1 *who sponsors one link*[10]*. Any deviation from such link sponsorship profile will incur a higher total link formation cost due to the convexity of the cost function. For example, if the lowest cost agent* L *sponsors four links instead of three links and the highest cost agent* H_1 *sponsors no link instead of one link, then the total cost of link formation increases from 930.96 to 932.04.*

[9] *Note that the payoff here assumes a cost function that is more general than that of Equation 1.*

[10] *Note that, of course, there are also other networks beside these eight networks that are efficient for the case of no decay. Specifically any network such that* L *sponsors one link, intermediate cost agents* M_1,M_2 *sponsor two links and high cost agents sponsor no links except* H_1 *who sponsors one link is efficient.*

FIGURE 2
EXAMPLE 2



A natural question that arises is how to narrow down such a large set of efficient networks. In what follows, by introducing the small decay assumption, I establish two algorithms that effectively serve this purpose. These two algorithms are simply a straightforward application of our main results as in Proposition 1[11]. while the formal statements of these two algorithms are included in the Appendix, their results are stated below. We first begin with some preliminary notations.

5. PRELIMINARY NOTATIONS

Let $n_i^S(g) = |g_i|$. That is, $n_i^S(g)$ is the number of links that agent i sponsors in g. We say that two networks g and g' are $LS-$*equivalent* if $n_i^S(g) = n_i^S(g')$ for every agent $i \in N$[12]. For example, all networks in Figure 2 are $LS-$*equivalent* because, as mentioned in the above Example 2, "agent L sponsors one link, intermediate cost agents M_1, M_2 sponsor two links and high cost agents sponsor no links except H_1 who sponsors one link." Next, we say that a network g' is *improved* from the network g if $I(g') > I(g)$ and g' and g are LS-equivalent. If $I(g') > I(g)$ we say that g' is *superior* to g. If $I(g') \geq I(g)$ for every $g \neq g'$ then we say that g' is *optimal* network.

11 These two algorithms are developed upon the suggestions of an anonymous reviewer, whom I would like to thank.

12 LS stands for link sponsorship.

_ Next, recall that a chain between i and j in a network g, denoted by $P_{ij}(g)$, is defined as a sequence of agents $\{\overline{i_0 i_1},...,\overline{i_{k-1} i_k}\}$ such that $i_0 = i, i_k = j$. A path is defined similarly, except that link sponsorship is one-directional. That is, a path from i to j is a sequence of agents $\{i_0 i_1,...,i_{k-1} i_k\}$ such that $i_0 = i, i_k = j$. A path from i to j in a network g is denoted by $P_{ij}(g)$. In a minimal network, a link ij is said to *point awa*y from an agent i' if ij is the last link in a chain between i' and j. A path is said to point away from an agent i' if every link in this path points away from i'. By the same analogy, a link ij is said to *point towards* an agent i' if ij is the last link in a chain between i' and i. A path is said to point towards i' if every link in this path points towards i'. Finally, in a minimal network a terminal agent is an agent that has precisely one link.

Next, we define the following sets of links. Let i^* be a best-informed agent in the network g. $L^{S,1}(g) = \{ij \in g \mid ij \text{ points towards } i^* \text{and} i, j \neq i^*\}$ and $L^{S,1}_{\mathrm{MOD}}(g) = \{ii^* \notin g \mid \text{every link } ij \in L^{S,1}(g) \text{become s} ii^*\}$. For example, in Figure 2 $L^{S,1}(g^I) = \{M_2H_2, M_1H_5\}$ and $L^{S,1}_{MOD}(g) = \{M_2L, M_1L\}$. Next, let $\hat{P}_{it}$ be a maximal multi-link path pointing away from a best -informed agent i^* to a terminal agent t. Let $L^{S,2}(g)$ be the set of all links that belong to such path(s) in g. We partition $L^{S,2}(g)$ into two sets: $L^{S,2^A}(g)$ is the set of all links ij such that ij is the first link on each path $\hat{P}_{it} \in L^{S,2}(g)$ and $L^{S,2^B}(g)$ is the rest of the links in $L^{S,2}(g)$. Two more sets are introduced as modifications of $L^{S,2^A}(g)$ and $L^{S,2^B}(g)$ as follows. $L^{S,2^A}_{MOD}(g) = \{i't \notin g \mid i'j' \in L^{S,2^A}(g) \, and \, i'j' \in \hat{P}_{i't}\}$. That is, for each link in $L^{S,2^A}(g)$ we fix the link sender but changes the link receiver to be a terminal agent. $L^{S,2^B}_{MOD}(g) = \{i'i^* \notin g \mid \text{there is an agent } j' \text{ such that } i'j' \in L^{S,2^B}(g)\}$. That is, for each link in $L^{S,2^B}(g)$ we fix the link sender but changes the link receiver to be the best-informed agent i^*. For example, consider the network $g^{I,2}$ in Figure 3. $L^{S,2}(g^{I,2})$ consists of two links - LH_1, H_1H_2. Hence, $L^{S,2^A}(g^{I,2}) = \{LH_1\}$, $L^{S,2^B}(g^{I,2}) = \{H_1H_2\}$, $L^{S,2^A}_{MOD}(g) = \{LH_2\}$, $L^{S,2^B}_{MOD}(g^{I,2}) = \{H_1L\}$.

We are now ready to state the first result of this section, which shows that Algorithm 1 always leads to an improved network.

Corollary 1. *Let the decay be small such that* $\sigma \in (\sigma_K, 1)$. *Consider a minimally connected network* g *such that* $L^{S,1}(g) \neq \varnothing$. *Let* $ij \in L^{S,1}(g)$. *Then the network* $g' = g - ij + ii^*$ *is improved from* g. *Thus, as in Algorithm 1 the final network* $g^* = (g \setminus L^{S,1}(g)) \cup L^{S,1}_{MOD}(g)$ *where* $L^{S,1}(g^*) = \varnothing$ *is improved from* g.

The proof of this corollary is straightforward and hence is omitted. Intuitively, in a network g Algorithm 1 replaces a link ij with a link ii^* where i^* is a best-informed agent. To understand why this leads to an improved network, observe that because we assume that in g i^* is a best-informed agent we know that in $g-ij$ i^* remains a best-informed agent. Thus, by Proposition 1 i^* is superior as a transmitter compared to i. It follows that $g-ij+ii^*$ is superior to $g-ij+ij=g$. The example below illustrate this intuition.

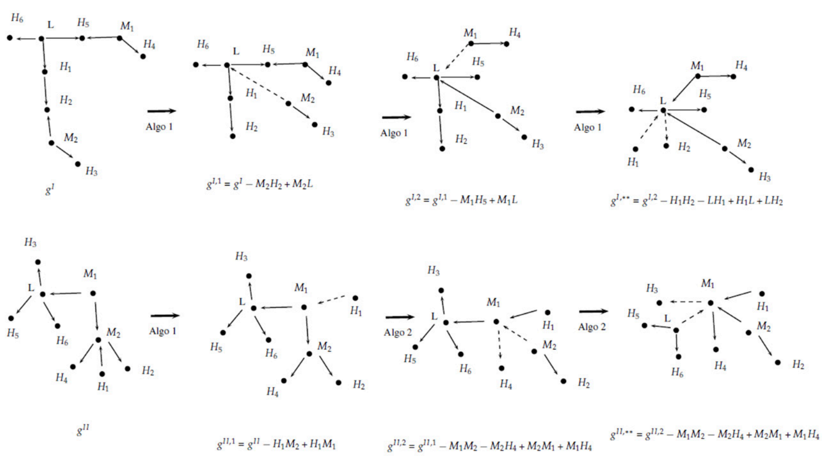
Example 3 *(Example of Algorithm 1)***.** *Figure 3 shows how the network g^I is improved into $g^{I,1}$, which is further improved into the final network $g^{I,2}$. It also shows how g^{II} is improved into the final network $g^{II,1}$.*

Next, we establish a result that guarantees that our Algorithm 2 always leads to an improved network. This result is a (non)straightforward corollary of our Proposition 1. The proof is relegated to the Appendix.

Corollary 2. *Let the decay be small such that $\sigma \in (\sigma_K, 1)$). In a minimally connected network g', let there be a two-link path consisting of three agents i', i and t where $it, i'i \in g$ and t is a terminal agent. Then, a modified network $g'' = g' - i'i - it + i't + ii^*$ is improved from g'. Thus, as in Algorithm 2 the final network $g^{**} = \left(g' \setminus L^{S,2}(g')\right) \cup L_{MOD}^{S,2^A}(g') \cup L_{MOD}^{S,2^B}(g')$ is improved from g'. Moreover, $L^{S,2}(g^{**}) = \varnothing$.*

FIGURE 3

AN EXAMPLE OF ALGORITHM 1 AND 2



Example 4. *Consider the network* $g^{I,2}$ *and Figure 3. Through one iteration of Algorithm 2 we achieved the network* $g^{I,**} = g^{I,2} - H_1H_2 - LH_1 + H_1L + LH_2$ ($g^{II,**} = g^{II,2} - M_1M_2 - M_2H_4 + M_2M_1 + M_1H_4$ *), and through two iterations of Algorithm 2 the network* $g^{II,1}$ *is transformed into* $g^{II,2}$ *and then* $g^{II,**}$. *Making use of Corollary 2we have* $\overline{I}\left(g^{I,**}\right) > \overline{I}\left(g^{I,2}\right)$ *and* $\overline{I}\left(g^{II,**}\right) > \overline{I}\left(g^{II,2}\right) > \overline{I}\left(g^{II,1}\right)$.

Finally, Algorithms 1 and 2 and the resultant Corollary 1 and 2 to establish the main result of this subsection below:

Corollary 3. *Let the decay be small such that* $\sigma \in \left(\sigma_K, 1\right)$ *). Any minimal network* g *such that* $L^{S,1}(g) \cup L^{S,2}(g) \neq \varnothing$ *can be improved through Algorithm 1 and 2 above. It can be improved into network* g^{**} *such that* $L^{S,1}\left(g^{**}\right) \cup L^{S,2}\left(g^{**}\right) = \varnothing$ *through the following procedure. First, use Algorithm 1 to obtain* $g^{*} = \left(g \setminus L^{S,1}(g)\right) \cup L^{S,1}_{MOD}(g)$. *Then use Algorithm 2 to obtain the final network* $g^{**} = \left(g^{*} \setminus L^{S,2}\left(g^{*}\right)\right) \cup L^{S,2^A}_{MOD}\left(g^{*}\right) \cup L^{S,2^B}_{MOD}\left(g^{*}\right)$. *This* g^{**} *has the following properties:*

4. *if an agent* i^{*} *is a best-informed agent in the initial network* g *then* i^{*} *is also a best-informed agent in* g^{**} .
5. *every chain from* i^{*} *to a terminal agent has at most two links. If a chain between* i^{*} *and a terminal agent is precisely one link, then either* i^{*} *or the terminal agent sponsor the link. If the chain between* i^{*} *and a terminal agent has precisely two links, then neither* i^{*} *nor the terminal agent act as a link sender.*
6. *since every chain from* i^{*} *to a terminal agent has at most two links,* g^{**} *has a diameter of at most* 4 .
7. *in* g^{**} *there are altogether* $K + n^{S}_{i^{*}}(g)$ *agents who are one-link away from* i^{*}, *where* K *is the number of all agents who are link senders except* i^{*} *in* g , *and the rest of the agents are two-link away from* i^{*} .

The proof of this corollary is straightforward and hence is omitted. Example 5 below provide an intuitive illustration for each of these properties.

Example 5. *Consider the network* g^{I} (g^{II}) *in Figure 3. Through two iterations of Algorithm 1 and then one iteration of Algorithm 2 we achieve the network* $g^{I,**}$. *Observe that in* g^{I} *agent* L *is the middle agent and hence the best-informed agent (See Footnote 7). For each of the iterations, observe that there is one more agent that becomes one link away from* L . *Thus,* L *remains a best-informed agent in the network throughout all iterations. This illustrates property (a) in the above Corollary 3. Next, observe that in* $g^{I,**}$ *each path from* L *to a terminal agent has at most two links. In particular, observe that* *the path from L to H_3 has precisely two links, both of which are sponsored by the agent M_2 who lies between the two agents. This illustrates properties (b) and (c) in the corollary above. Finally, observe that in $g^{I,**}$ there are six agents that are one link away from L . Three of these six (five) agents are accessed by i^* , while the other three agents sponsor themselves. This illustrates properties (d) in the corollary above.*

Importantly, the last property in the above Corollary 3 also necessitates that the final network g^{**} achieved through Algorithm 1 and 2 is not necessarily optimal within the class of all $LS-$ equivalent networks[13]. We state this observation as a remark and then provide an example below.

Remark 3. *Let the decay be small such that $\sigma \in (\sigma_K, 1)$). As a corollary of the property (d) in 3, consider two minimally connected $LS-$ equivalent networks g^1 and g^2 such that $L^{S,1}(g) \cup L^{S,2}(g) \neq \varnothing$. Let $i^{*,1}$ and $i^{*,2}$ be the best-informed agent in g^1 and g^2 respectively and let $g^{1,**}$ and $g^{2,**}$ be the final networks achieved through Algorithm 1 and 2. If $n^S_{i^{*,1}}(g^1) > n^S_{i^{*,2}}(g^2)$ then $g^{1,**}$ is superior to $g^{2,**}$. Thus, within the class of LS-equivalent network, only the final network g^{**} such that the best-informed agent is a largest sponsor is a unique optimal network.*

Example 6 below provides an intuitive illustration of Remark 3above.

Example 6. *Consider networks g^I and g^{II} in Figure 3. Observe that g^I and g^{II} are $LS-$equivalent. Next, observe that L and M_1 are the best-informed agents in g^I and g^{II} respectively but $n^S_L(g^I) = 3 > n^S_{M_1}(g^{II}) = 2$* [14]. *Next, consider the two final networks $g^{I,**}$ and $g^{II,**}$ improved from g^I and g^{II} respectively. Observe that $n^1_L(g^{I,**}) = 6 > n^1_{M_1}(g^{II,**}) = 5$ while $n^2_L(g^{I,**}) = 2 > n^2_{M_1}(g^{II,**}) = 3$. Thus, in $g^{I,**}$ there are more agents who are of distant 2 from each other than in $g^{II,**}$, yet there are less agents who are of distant 3 and 4 from each other than in $g^{II,**}$. We conclude that $g^{I,**}$ is superior to $g^{II,**}$.*

Finally, we apply our Algorithms 1 and 2 to narrow down and precisely identify the set of efficient networks in Example 1 as follows:

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[13] In the sense that there exists another network g^{***} that is $LS-$ equivalent to g^{**} such that $\bar{I}(g^{***}) > \bar{I}(g^{**})$.

[14] *For each agent i in a network g, we denote the set of agents that are of k-links away from i as $N^k_i(g)$. Naturally, $n^k_i(g) = |N^k_i(g)|$ is number of agents that are of k -links away from i .*

Example 7. *As a continuation of Example 2, first recall that all networks in Figure 2 are efficient networks if there is no decay. Now, if we assume small decay by setting* $\sigma = 0.999$*, we can partially rank these networks and hence narrow down the set of efficient networks as follows. First, making use of Algorithm 1 and Algorithm 2 as shown in Example 3 and 4 we conclude that* $\bar{I}(g^{I}) < \bar{I}(g^{I,1}) < \bar{I}(g^{I,2}) < \bar{I}(g^{I,**})$ *and* $\bar{I}(g^{II}) < \bar{I}(g^{II,1}) < \bar{I}(g^{II,2}) < \bar{I}(g^{II,**})$*. Second, because we assume player heterogeneity in link formation cost and that all these networks are LS-equivalent, we know that all these networks have the same total link formation cost. Combining these two reasons, we conclude that* g^{II} *is dominated by* $g^{II,1}$*, which is dominated by* $g^{II,2}$*, which is dominated by* $g^{II,**}$*. Thus, among eight networks that are efficient when there is no decay, six are eliminated as candidates for efficient networks if small decay is assumed. Only* $g^{I,**}$ *and* $g^{II,**}$ *remain candidates for an efficient network.*

Next, we answer the natural question that arises: which network $g^{I,**}$ *or* $g^{II,**}$ *- dominates the other? Recall from Example 5that* $\bar{I}(g^{I,**}) > \bar{I}(g^{II,**})$ *. Following the same line of reasoning as in the above paragraph, we conclude that* $g^{I,**}$ *dominates* $g^{II,**}$*. Indeed, recall from Remark 3 that within the class of LS-equivalent network,* $g^{II,**}$ *is a unique optimal network because its best-informed agent* L *is the largest sponsor. Furthermore, recall that we set our decay to be very small, that is,* $\sigma = 0.999$*. Thus, any network that does not dominate* $g^{I,**}$ *in the case of no decay also does not dominate* $g^{I,**}$ *for* $\sigma = 0.999$*. It follows that* $g^{I,**}$ *is a unique efficient network.*

Observe how useful our Algorithms 1 and 2 are. From eight efficient networks in case of no decay as in Example 2, this set is narrowed to just one efficient network by assuming small decay and applying Algorithms 1 and 2.

6. Conclusions

In this note, I show that in a Nash network every link receiver is an efficient transmitter of information in the simple model of two-way flow network with nonrival information pioneered by Bala and Goyal (2000b) if small decay of information is assumed. This results in an insight that a strategic decision of self-interested agents to form links can, in part, leads to a socially desirable outcome. In Section 4, I also show how this result can be applied to refine and resolve the technical difficulties in some existing literature in terms of characterization of efficient networks.

It is important to keep in mind, however, that this note follows the convention in the literature by assuming that the benefit of each agent is precisely the total information he receives in the network. Consequently, how a more general form of benefit function could alter the characteristic of efficient network and the main result of this note becomes a future research question to explore. In addition, another open question is of whether the result of this note can be extended to a more general case such that any level of decay is assumed.

Appendix

Algorithms

Algorithm 1.

1. Initialization:
a. Begin with the network g *such that* $L^{S,1}(g) \neq \varnothing$.
2. Selection:
a. Choose any link $ij \in L^{S,1}(g)$.
3. Modification:
a. Obtain a modified network g' *by performing the following operations on* g*:*
i. Remove the link ij.
ii. Add the link ii^*.
b. Thus, $g' = g - ij + ii^*$. *Note that* $L^{S,1}(g') = L^{S,1}(g) \setminus \{ij\}$
4. Check and Repeat:
a. If $L^{S,1}(g') \neq \varnothing$ *:*
i. Set $g = g'$.
ii. Repeat steps 2 and 3 with the updated network s .
5. Termination:
a. Continue the process until $L^{S,1}(g') = \varnothing$.
b. Denote the final network as g^*. *Note that* $g^* = \left(g \setminus L^{S,1}(g)\right) \cup L^{S,1}_{MOD}(g)$, *where* g *is the initial network.*

Algorithm 2.

1. Initialization:
a. Begin with the network $g^{`}$ *such that* $L^{S,2}(g^{`}) \neq \varnothing$.
2. Selection:
a. Choose two links $i_0 i_1$ *and* $i_1 i_2$ *in* $L^{S,2}(g^{`})$ *such that* i_2 *is a terminal agent.*
3. Modification:
a. Obtain a modified network $g^{``}$ *by performing the following operations on* $g^{`}$*:*
i. Remove the link $i_0 i_1$ *and* $i_1 i_2$.
ii. Add the link $i_0 i_2$ *and* $i_1 i^*$.
b. Thus, $g^{``} = g^{`} - i_0, i_1 - i_1 i_2 + i_0, i_2 + i_1 i^*$.
4. Check and Repeat:
a. If $L^{S,2}(g^{``}) \neq \varnothing$ *:*
i. Set $g^{``} = g^{`}$.

ii. Repeat steps 2 and 3 with the updated network $g^{`}$.

5. Termination:

a. Continue the process until $L^{S,2}(g^{`})=\varnothing$.

b. Denote the final network as g^{**}.

USEFUL LEMMATA

Proof 1 (Proof of Lemma 1). *According to Proposition 1 and Lemma 1 in Unlu (2018), in the absence of decay, every nonempty efficient network is minimally connected. Since the aggregate payoff* $W(g)$ *is continuous in* σ *, it follows that for any* $c>0$*, there exists* $\sigma_{M'}<1$ *such that if* $\sigma>\sigma_{M'}$*, then every efficient network is minimally connected.*

Next, recall from Lemma 4 in De Jaegher and Kamphorst (2015) that for any $c>0$ *and* $n\geq 4$*, there exists* $\sigma_M<1$ *such that if* $\sigma>\sigma_M$*, then every Nash network is minimal. Also, recall from Lemmas 5 and 6 in De Jaegher and Kamphorst (2015) that every nonempty minimal Nash network is connected.*

Hence, there exists $\sigma_K<1$*, where* $\sigma_K=\max(\sigma_M,\sigma_{M'})$*, such that if* $\sigma>\sigma_K$*, then every efficient network and every Nash network is minimally connected.*

Proof 2 (Proof of Lemma 2). *First, consider* $i\in N'$*. Observe that in* g' i *receives information of agent in* g'' *via the agent* $x\in N'$ *who, in turn, receives from the* $y\in N''$ *with whom he is one-link away because* $\overline{xy}\in g$*. This fact implies that* $\sum_{l\in N''}I_{xl}(g)=\sigma I_y(g'')$ *and* $\sum_{l\in N''}I_{il}(g)=\sigma^{d_{ix}(g')}\sigma I_y(g'')$*. Next, denote* $\overline{I}_{y\to x}(g)$ *- the total information that all agents in* g' *receives from* g'' *via the link* $\overline{xy}$*. That is,*

$$\overline{I}_{y\to x}(g)=\sum_{i\in N'}\left(\sum_{l\in N''}I_{il}(g)\right)=\sum_{i\in N'}\left(\sigma^{d_{ix}(g')}\sigma I_y(g'')\right)=\sigma I_y(g'')\sum_{i\in N'}\left(\sigma^{d_{ix}(g')}\right)=\sigma I_y(g'')I_x(g').$$

On the other hand, we know that the total information that all agents in g' *exchange with each other is* $\overline{I}(g')$*. This fact and the above expression lead to*

$$\sum_{i\in N'}I_i(g)=\overline{I}(g')+\overline{I}_{y\to x}(g)=\overline{I}(g')+\sigma I_y(g')I_x(g'').$$

This completes the first part of this lemma. The second part of this lemma follows the same analogy so that;

$$\sum_{i\in N''}I_i(g)=\overline{I}(g'')+\sigma I_y(g')I_x(g'').$$

Lastly, observe that part (iii) is simply due to the fact that $\sum_{i\in N}I_i(g)=\sum_{i\in N'}I_i(g)+\sum_{i\in N''}I_i(g)$*. This completes our proof.*

Proof Of Proposition 1

Proof 3. First, by Lemma 1 we know that if decay is sufficiently small then all Nash networks and efficient networks are minimally connected. Let $g' = D^x_{\overline{xy}}, g'' = D^y_{\overline{xy}}, N' = N^x_{\overline{xy}}$ and $N'' = N^y_{\overline{xy}}$ so that $g = g' \oplus_{\overline{xy}} g''$ and $g - \overline{xy} + \overline{xj'} = g' \oplus_{\overline{xj'}} g''$ and $g - \overline{xy} + \overline{xj''} = g' \oplus_{\overline{xj''}} g''$ for any $j', j'' \in N^y_{\overline{xy}}$. Using Equation in (3) of Lemma 2 we know that $\overline{I}\left(g - \overline{xy} + \overline{xj'}\right) \geq \overline{I}\left(g - \overline{xy} + \overline{xj''}\right)$ if and only if $\overline{I}_{j'}\left(g''\right) = \overline{I}_{j'}\left(D^y_{\overline{xy}}\right) \geq \overline{I}_{j''}\left(g''\right) = \overline{I}_{j''}\left(D^y_{\overline{xy}}\right)$. Hence, j' is superior to j'' as a transmitter if and only if j' is better informed than j'' and j' is an efficient transmitter if and only if j' is best informed with respect to the link $\overline{xy}$.

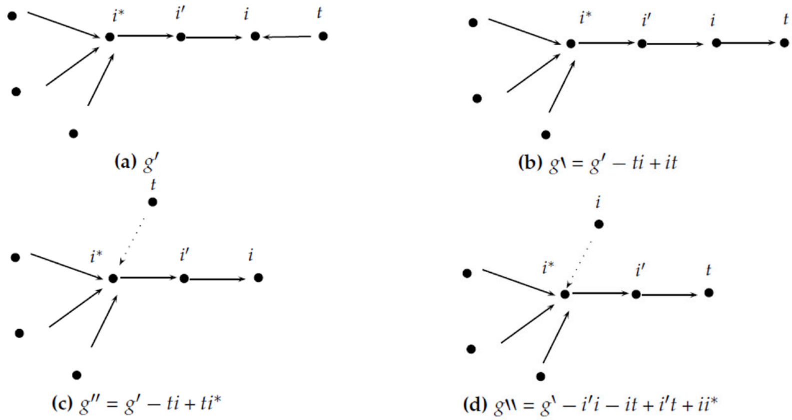
Finally, to prove (ii), recall Remark 1 in De Jaegher and Kamphorst (2015) that if a link $ij \in g$ and g is Nash then j is a best-informed agent with respect to ij.

Proof 4 (Proof of Corollary 2). *First, consider the network g' as in Figure 4a, which has i^* as a best-informed agent and a terminal agent t who sponsors himself by accessing another agent i, who receives a link from another agent i'. Observe that if we modify g' into $g^{`} = g' - ti + it$ as in Figure 4b then $g^{`}$ has a two-link path consisting of three agents i', i and t where $it, i'i \in g$ and t is a terminal agent, which is precisely what this lemma assumes. Observe further that by our construction g' and $g^{`}$ share the same architecture. Thus, $\overline{I}(g') = \overline{I}\left(g^{`}\right)$.*

Next, we modify g' and $g^{`}$ into $g'' = g' - ti + ti^*$ and $g^{``} = g^{`} - i'i - it + i't + ti^*$ as in Figures 4c and 4d. Again, by our construction, $g''$ and $g^{``}$ share the same architecture. Thus, $\overline{I}\left(g''\right) = \overline{I}\left(g^{``}\right)$.

Next, because $g'' = g' - ti + ti^*$ and i^* is a best-informed agent by Proposition 1 we have $\overline{I}(g'') > \overline{I}(g')$. Finally, making use of the last sentences of the above two paragraphs we conclude that $\overline{I}(g'') = \overline{I}\left(g^{``}\right) > \overline{I}\left(g^{`}\right) = \overline{I}(g')$. This completes our proof.

FIGURE 4
OUR NETWORKS AS IN THE PROOF OF COROLLARY 2



References

Bala, V., and Goyal, S. (2000a). "A Strategic Analysis of Network Reliability," Review of Economic Design, Vol. 5(3); 205–228.

Bala, V., and Goyal, S. (2000b). "A Noncooperative Model of Network Formation," Econometrica, Vol. 68(5); 1181–1229.

Bloch, F., and Dutta, B. (2009). "Communication Networks with Endogenous Link Strength," Games and Economic Behavior, Vol. 66(1); 39–56.

Bloch, F., and Jackson, M.O. (2007). "The Formation of Networks with Transfers Among Players," Journal of Economic Theory, Vol. 133(1); 83–110.

Breitmoser, Y., and Vorjohann, P. (2013). "Efficient Structure of Noisy Communication Networks," Mathematical Social Sciences, Vol. 66(3); 396–409.

Charoensook, B. (2020). "On the Interaction Between Small Decay, Agent Heterogeneity and Diameter of Minimal Strict Nash Networks in Two-Way Flow Model," Annals of Economics and Finance, Vol. 21; 331–361.

De Jaegher, K., and Kamphorst, J. (2015). "Minimal Two-Way Flow Networks with Small Decay," Journal of Economic Behavior & Organization, Vol. 109; 217–239.

Dutta, B., Ghosal, S., and Ray, D. (2005). "Farsighted Network Formation," Journal of Economic Theory, Vol. 122(2); 143–164.

Hojman, D.A., and Szeidl, A. (2008). "Core and Periphery in Networks," Journal of Economic Theory, Vol. 139(1); 295–309.

Hoyer, B., and De Jaegher, K. (2023). "Network Disruption and the Common-Enemy Effect," International Journal of Game Theory, Vol. 52(1); 117–155.

Jackson, M.O. (2008). Social and Economic Networks. Princeton University Press, Princeton, NJ, USA.

Jackson, M.O., and Wolinsky, A. (1996). "A Strategic Model of Social and Economic Networks," Journal of Economic Theory, Vol. 71(1); 44–74.

Olaizola, N., and Valenciano, F. (2021). "Efficiency and Stability in the Connections Model with Heterogeneous Nodes," Journal of Economic Behavior & Organization, Vol. 189; 490–503.

Unlu, E. (2018). "Efficient Networks in Models with Player and Partner Heterogeneity," The Manchester School, Vol. 86(1); 100–118.